

Calculator and mobile phones are NOT allowed.

Answer the following questions. Each sub(question) is worth 4 points

Q1. Show that $f(x) = \sqrt{1 + \ln(x^2 + 1)}$, $x \geq 0$ is one-to-one. Find $f^{-1}(x)$ and state its domain.

Q2. Evaluate $\lim_{x \rightarrow 0^+} \left(\frac{1}{\tan x} - \frac{1}{x} \right)$

Q3. Evaluate the following integrals:

(a) $\int (\ln x)^2 dx$

(b) $\int \frac{e^{2x}}{\sqrt{e^{4x} + 2e^{2x} + 2}} dx$

(c) $\int \frac{(2\sin x - 3)\cos x}{\sin^2 x + 3\sin x + 2} dx$

Q4. Determine whether the integral $\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}} dx$ converges or diverges, if it converges, find its value.

Q5. Find the length of the curve C given by

$$x = t^2 \cos t, \quad y = t^2 \sin t \quad ; \quad 0 \leq t \leq 2\pi.$$

Q6. Find the area of the region that is inside the circle $r = 9\cos\theta$ and outside the cardioid $r = 3 + 3\cos\theta$.

Q7. If θ is the angle between nonzero vectors \mathbf{a} and \mathbf{b} , then prove that

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos\theta$$

Q8. Find the equation of the plane through the point $P(6, 2, -1)$ and perpendicular to the line of intersection of the planes $4x - 3y + 2z + 5 = 0$ and $3x + 2y - z + 11 = 0$.